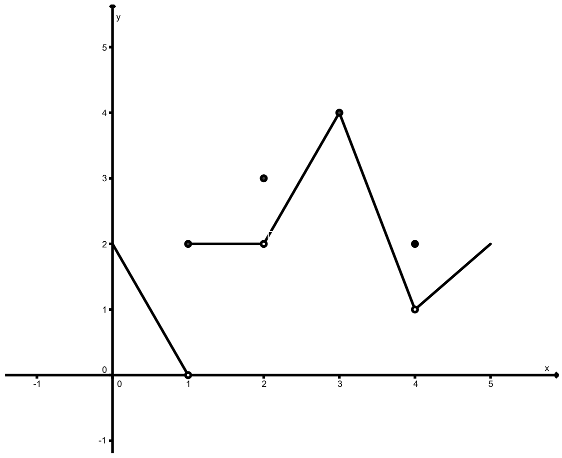
Difference Between Function Value and Limit

In this handout we ask if $\lim_{x \to a} f(x)$ is always equal to $f(a)$? To answer this question, consider the graph of the function $f(x)$ below.



We will examine the graph in stages.
When x=1:

What is $f(1)$? In order to do this we need to go to an x value of 1 along the x axis and go vertically until you hit the graph. Then go across to find the corresponding y value. If we do this in the graph above we see that

$$f(1) = 2$$

Now suppose we need to find $\lim_{x \to 1^-} f(x)$. In other words, if we were to approach an x value of 1 from the left (numbers smaller than 1), what would the y value approach? Examining the graph we see that

$$\lim_{x \to 1^-} f(x) = 0$$

Next suppose we need to find $\lim_{x \to 1^+} f(x)$. In other words, if we were to approach an x value of 1 from the right (through numbers bigger than 1), what would the y value approach? Once again, examining the graph we see that

$$\lim_{x \to 1^+} f(x) = 2$$

Material developed by the Department of Mathematics & Statistics, N.U.I. Maynooth and supported by the NDLR (www.ndlr.com).

In situations like this, where the limit from the left does not equal the limit from the right, we say that

$$\lim_{x \to 1} f(x) \text{ does not exist.}$$

Note here that $f(1) = 2$ but $\lim_{x \to 1} f(x)$ does not exist.

When x=2:

Examining the graph, and using the same methods as above we can say that:

$$\begin{aligned} f(2) &= 3 \\ \lim_{x \to 2^-} f(x) &= 2 \\ \lim_{x \to 2^+} f(x) &= 2 \end{aligned}$$

In this situation we see that the limit from the left equals the limit from the right. We can therefore say

$$\lim_{x \to 2} f(x) = 2$$

Notice that $f(2) \neq \lim_{x \to 2} f(x)$.

When x=3:

Examining the graph we see:

$$\begin{aligned} f(3) &= 4 \\ \lim_{x \to 3^-} f(x) &= 4 \\ \lim_{x \to 3^+} f(x) &= 4 \end{aligned}$$

In this situation we see that the limit from the left equals the limit from the right. We can therefore say

$$\lim_{x \to 3} f(x) = 4$$

Notice that $f(3) = \lim_{x \to 3} f(x)$. In situations like this, where $\lim_{x \to a} f(x) = f(a)$, we say that $f(x)$ is continuous at $x = a$.

For practice, using the graph above, find:
(a)$f(4)$ (b)$\lim_{x \to 4^-} f(x)$ (c)$\lim_{x \to 4^+} f(x)$ (d) $\lim_{x \to 4} f(x)$

Solutions
(a) 2 (b) 1 (c) 1 (d) 1