GEOMETRIC SERIES I (OF II)

Definition of a Geometric Series

Let x be a real number. Then we can consider the following sequence of real numbers

$$1, \quad x, \quad x^2, \quad x^3, \quad x^4, \quad x^5, \quad \ldots$$

This sequence can be expressed in a more compact form as

$$(x^n)_{n\geq 0}.$$

For every sequence there is the corresponding series, or sequence of partial sums. In our case the corresponding series is expressed by

$$\sum_{n=0}^{\infty} x^n$$

and we call this series the **geometric series**. The elements of the geometric series are:

$$\begin{aligned} s_0 &= 1 \\ s_1 &= 1 + x \\ s_2 &= 1 + x + x^2 \\ s_3 &= 1 + x + x^2 + x^3 \\ &\vdots \\ s_k &= \sum_{n=0}^{k} x^n, \quad \text{for } k \geq 0 \end{aligned}$$

Examples of Geometric Series

For every real number x we get a different geometric series. That means a geometric series is uniquely determined by the value of x.

Example 1.1: Let $x = 1$. Then the sequence $(x^n)_{n\geq 0}$ consists of the following elements

$$x^0 = 1, \quad x^1 = 1, \quad x^2 = 1, \quad x^3 = 1, \quad x^4 = 1, \ldots$$

Material developed by the Department of Mathematics & Statistics, NUIM and supported by www.ndlr.com.

That is, all elements are equal to one. Now the corresponding geometric series

$$\sum_{n=0}^{\infty} 1$$

has the elements

$$s_0 = 1, \quad s_1 = 2, \quad s_2 = 3, \quad s_3 = 4, \quad s_4 = 5, \quad \ldots$$

Example 1.2 If $x = -2$, then the sequence $(x^n)_{n \geq 0}$ consists of the elements

$$x^0 = 1, \quad x^1 = -2, \quad x^2 = 4, \quad x^3 = -8, \quad x^4 = 16, \ldots$$

and the corresponding geometric series

$$\sum_{n=0}^{\infty} (-2)^n$$

has the elements

$$s_0 = 1, \quad s_1 = -1, \quad s_2 = 3, \quad s_3 = -5, \quad s_4 = 11, \quad \ldots$$

Example 1.3 If $x = \frac{1}{2}$, then the sequence $(x^n)_{n \geq 0}$ consists of the elements

$$x^0 = 1, \quad x^1 = \frac{1}{2}, \quad x^2 = \frac{1}{4}, \quad x^3 = \frac{1}{8}, \quad x^4 = \frac{1}{16}, \ldots$$

and the corresponding geometric series

$$\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n$$

has the elements

$$s_0 = 1, \quad s_1 = \frac{3}{2}, \quad s_2 = \frac{7}{4}, \quad s_3 = \frac{15}{8}, \quad s_4 = \frac{31}{16}, \quad \ldots$$

Remark 1.4: See handout Geometric Series II for a discussion/ explanation of the convergence of the geometric series.