

Introduction to Limits

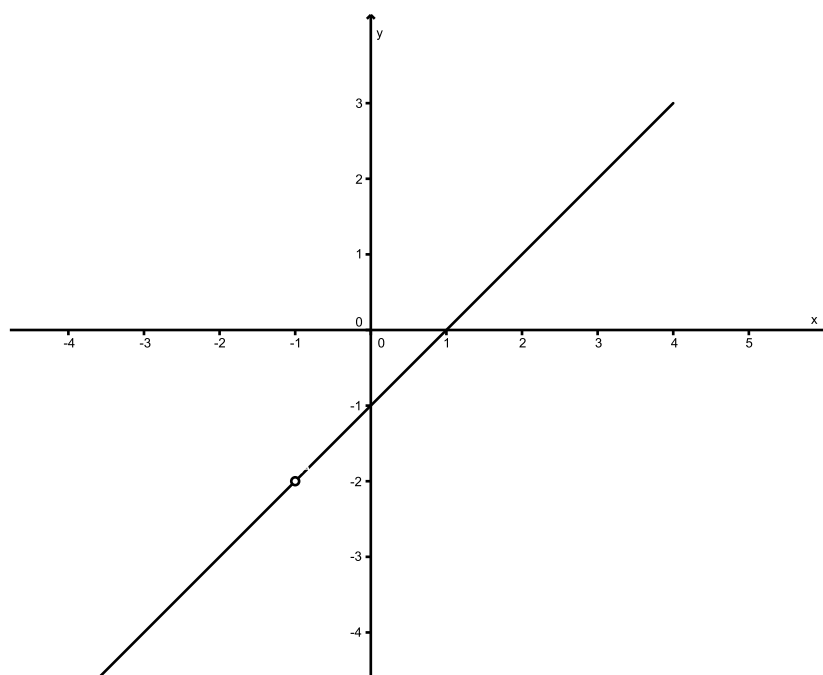
Limits are an extremely useful tool which are used in a wide range of areas in mathematics. For example limits are essential in determining the continuity of functions, sketching graphs, determining the convergence and divergence of sequences or series. Understanding limits can give you an insight into these and many other areas of mathematics.

We write $\lim_{x \rightarrow a} f(x) = L$ for a and $L \in \mathbb{R}$, if we can make the values of $f(x)$ as close as we like to L by taking x sufficiently close to a .

Consider the function

$$f(x) = \frac{x^2 - 1}{x + 1}$$

and its graph below.



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Notice that $f(-1)$ is undefined, but when $x \neq -1$ we can say that

$$\begin{aligned} f(x) &= \frac{x^2 - 1}{x + 1} \\ &= \frac{(x + 1)(x - 1)}{x + 1} \\ &= x - 1. \end{aligned}$$

By taking values of x very close to -1 , we can make $f(x)$ as close as we like to -2 . In this case we write that

$$\lim_{x \rightarrow -1} f(x) = -2.$$

Limits of this form will be very important when calculating derivatives.