Limit Examples 1

In this handout we will look at two elementary limit problems. Problems of the type below, while elementary, are very important and occur frequently in calculus.

Example 1
Find

$$\lim_{x \to 2} x^2 + 3x - 1.$$

In this problem we are looking to see what happens to the value of the function $x^2 + 3x - 1$ as x gets very close to 2. We also notice that $x^2 + 3x - 1$ is a very familiar form to us, it's a polynomial and the $\lim_{x \to a} p(x) = p(a)$ for all polynomials p. This tells us that to do this problem, we just need to substitute $x = 2$ into the function $x^2 + 3x - 1$. We have:

$$\begin{aligned} \lim_{x \to 2} x^2 + 3x - 1 &= (2)^2 + 3(2) - 1 \\ &= 4 + 6 - 1 \\ &= 9. \end{aligned}$$

We have found that the $\lim_{x \to 2} x^2 + 3x - 1 = 9$.

Example 2
Find

$$\lim_{x \to 1} \frac{x^2 - x + 2}{x^2 + 1}.$$

In this case we have a rational function, i.e. one polynomial over another polynomial. Before we calculate the limit, we must check that substituting $x = 1$ into the function $\frac{x^2 - x + 2}{x^2 + 1}$ does not cause it to become undefined. We know this could happen if the denominator (the part under the line) is 0 when $x = 1$ is substituted into it. It is simple to check that when $x = 1$ substituted into $x^2 + 1$ is not equal to 0 but equal to 2. We use rule 6 from the properties of limits handout to do this problem. So

$$\lim_{x \to 1} \frac{x^2 - x + 2}{x^2 + 1} = \frac{(1)^2 - (1) + 2}{(1)^2 + 1}$$

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$$
\begin{aligned}
&= \frac{1-1+2}{1+1} \\
&= \frac{2}{2} \\
&= 1.
\end{aligned}
$$

Recall from the handout on the "Properties of Limits" that $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ provided these limits exist and $\lim_{x\to a} g(x) \neq 0$. This leads to the question of what we would do if $\lim_{x\to a} g(x) = 0$? For example if we had to find the $\lim_{x\to 0} \frac{1}{x}$ or $\lim_{x\to 0} \frac{56}{x^2}$. You should see in either of these cases it is impossible to calculate the limit immediately. The various techniques used to deal with this problem are dealt with in other handouts in this series.

Try the following exercises for practice:

(a)

$$\lim_{x\to 2} 3x^2 + 6x - 2$$

(b)

$$\lim_{x\to \frac{\pi}{2}} \frac{2\sin x}{x}$$

(c)

$$\lim_{x\to 3} \frac{x-1}{x^2+3}$$

(d)

$$\lim_{x\to 1} \frac{2x^2-2x+3}{x^2+x+3}$$

(e)

$$\lim_{x\to 0} \frac{3x^2+6x+7}{\cos x}$$

Solutions:

(a) 22 (b) $\frac{4}{\pi}$ (c) $\frac{1}{6}$ (d) $\frac{3}{5}$ (e) 7