

Engineering Maths

First Aid Kit

2.2

Negative and fractional powers

Introduction

Sometimes it is useful to use negative and fractional powers. These are explained on this leaflet.

1. Negative powers

Sometimes you will meet a number raised to a negative power. This is interpreted as follows:

$$a^{-m} = \frac{1}{a^m}$$

This can be rearranged into the alternative form:

$$a^m = \frac{1}{a^{-m}}$$

Example

$$3^{-2} = \frac{1}{3^2}, \quad \frac{1}{5^{-2}} = 5^2, \quad x^{-1} = \frac{1}{x^1} = \frac{1}{x}, \quad x^{-2} = \frac{1}{x^2}, \quad 2^{-5} = \frac{1}{2^5} = \frac{1}{32}$$

Exercises

1. Write the following using only positive powers:

a) $\frac{1}{x^{-6}}$, b) x^{-12} , c) t^{-3} , d) $\frac{1}{4^{-3}}$, e) 5^{-17} .

2. Without using a calculator evaluate a) 2^{-3} , b) 3^{-2} , c) $\frac{1}{4^{-2}}$, d) $\frac{1}{2^{-3}}$, e) $\frac{1}{4^{-3}}$.

Answers

1. a) x^6 , b) $\frac{1}{x^{12}}$, c) $\frac{1}{t^3}$, d) 4^3 , e) $\frac{1}{5^{17}}$.

2. a) $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$, b) $\frac{1}{9}$, c) 16, d) 32, e) 64.

2. Fractional powers

To understand fractional powers you first need to have an understanding of roots, and in particular square roots and cube roots. If necessary you should consult leaflet 1.2 *Powers and Roots*.

When a number is raised to a fractional power this is interpreted as follows:

$$a^{1/n} = \sqrt[n]{a}$$

So,

$a^{1/2}$ is a square root of a

$a^{1/3}$ is the cube root of a

$a^{1/4}$ is a fourth root of a

Example

$$3^{1/2} = \sqrt{3}, \quad 27^{1/3} = \sqrt[3]{27} \text{ or } 3, \quad 32^{1/5} = \sqrt[5]{32} = 2, \\ 64^{1/3} = \sqrt[3]{64} = 4, \quad 81^{1/4} = \sqrt[4]{81} = 3$$

Fractional powers are useful when we need to calculate roots using a scientific calculator. For example to find $\sqrt[7]{38}$ we rewrite this as $38^{1/7}$ which can be evaluated using a scientific calculator. You may need to check your calculator manual to find the precise way of doing this, probably with the buttons x^y or $x^{1/y}$.

Check that you are using your calculator correctly by confirming that

$$38^{1/7} = 1.6814 \quad (4 \text{ dp})$$

More generally $a^{m/n}$ means $\sqrt[n]{a^m}$, or equivalently $(\sqrt[n]{a})^m$.

$$a^{m/n} = \sqrt[n]{a^m} \text{ or equivalently } (\sqrt[n]{a})^m$$

Example

$$8^{2/3} = (\sqrt[3]{8})^2 = 2^2 = 4, \quad \text{and} \quad 32^{3/5} = (\sqrt[5]{32})^3 = 2^3 = 8$$

Exercises

1. Use a calculator to find a) $\sqrt[5]{96}$, b) $\sqrt[4]{32}$.
2. Without using a calculator, evaluate a) $4^{3/2}$, b) $27^{2/3}$.
3. Use the third law of indices to show that

$$a^{m/n} = \sqrt[n]{a^m}$$

and equivalently

$$a^{m/n} = (\sqrt[n]{a})^m$$

Answers

1. a) 2.4915, b) 2.3784. 2. a) $4^{3/2} = 8$, b) $27^{2/3} = 9$.